10.3 Solving Radical Equations

Essential Question How can you solve an equation that contains square roots?

EXPLORATION 1 Analyzing a Free-Falling Object

Work with a partner. The table shows the time t (in seconds) that it takes a free-falling object (with no air resistance) to fall d feet.

d (feet)	t (seconds)
0	0.00
32	1.41
64	2.00
96	2.45
128	2.83
160	3.16
192	3.46
224	3.74
256	4.00
288	4.24
320	4.47

MODELING WITH MATHEMATICS

To be proficient in math, you need to routinely interpret your mathematical results in the context of the situation and reflect on whether the results make sense.

a. Use the data in the table to sketch the graph of t as a function of d. Use the coordinate plane below.

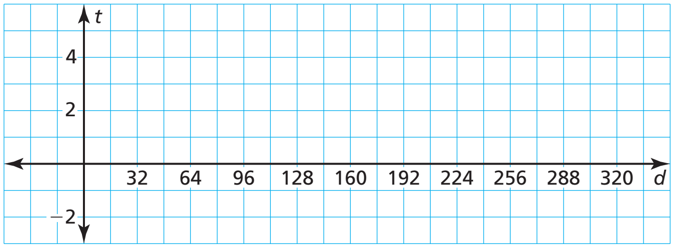
b. Use your graph to estimate the time it takes the object to fall 240 feet.

c. The relationship between d and t is given by the function

$$t = \sqrt{\frac{d}{16}}.$$

Use this function to check your estimate in part (b).

d. It takes 5 seconds for the object to hit the ground. How far did it fall? Explain your reasoning.



EXPLORATION 2 Solving a Square Root Equation

Work with a partner. The speed s (in feet per second) of the free-falling object in Exploration 1 is given by the function

$$s = \sqrt{64d}.$$

Find the distance the object has fallen when it reaches each speed.

a. $s = 8$ ft/sec **b.** $s = 16$ ft/sec **c.** $s = 24$ ft/sec

Communicate Your Answer

3. How can you solve an equation that contains square roots?

4. Use your answer to Question 3 to solve each equation.

a. $5 = \sqrt{x + 20}$ **b.** $4 = \sqrt{x - 18}$

c. $\sqrt{x} + 2 = 3$ **d.** $-3 = -2\sqrt{x}$

10.3 Lesson

Core Vocabulary

radical equation, *p. 560*

Previous
radical
radical expression
extraneous solution

What You Will Learn

- Solve radical equations.
- Identify extraneous solutions.
- Solve real-life problems involving radical equations.

Solving Radical Equations

A **radical equation** is an equation that contains a radical expression with a variable in the radicand. To solve a radical equation involving a square root, first use properties of equality to isolate the radical on one side of the equation. Then use the following property to eliminate the radical and solve for the variable.

Core Concept

Squaring Each Side of an Equation

Words If two expressions are equal, then their squares are also equal.

Algebra If $a = b$, then $a^2 = b^2$.

EXAMPLE 1 Solving Radical Equations

Solve each equation.

a. $\sqrt{x} + 5 = 13$ **b.** $3 - \sqrt{x} = 0$

SOLUTION

a. $\sqrt{x} + 5 = 13$ Write the equation.

$\sqrt{x} = 8$ Subtract 5 from each side.

$(\sqrt{x})^2 = 8^2$ Square each side of the equation.

$x = 64$ Simplify.

The solution is $x = 64$.

Check

$\sqrt{x} + 5 = 13$

$\sqrt{64} + 5 \stackrel{?}{=} 13$

$8 + 5 \stackrel{?}{=} 13$

$13 = 13$ ✓

b. $3 - \sqrt{x} = 0$ Write the equation.

$3 = \sqrt{x}$ Add $\sqrt{x}$ to each side.

$3^2 = (\sqrt{x})^2$ Square each side of the equation.

$9 = x$ Simplify.

The solution is $x = 9$.

Check

$3 - \sqrt{x} = 0$

$3 - \sqrt{9} \stackrel{?}{=} 0$

$3 - 3 \stackrel{?}{=} 0$

$0 = 0$ ✓

Monitoring Progress

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Solve the equation. Check your solution.

1. $\sqrt{x} = 6$

2. $\sqrt{x} - 7 = 3$

3. $\sqrt{y} + 15 = 22$

4. $1 - \sqrt{c} = -2$

Check

$4\sqrt{x+2}+3=19$

$4\sqrt{14+2}+3 \stackrel{?}{=} 19$

$4\sqrt{16}+3 \stackrel{?}{=} 19$

$4(4)+3 \stackrel{?}{=} 19$

$19 = 19$ ✓

EXAMPLE 2 Solving a Radical Equation

$4\sqrt{x+2}+3=19$	Original equation
$4\sqrt{x+2}=16$	Subtract 3 from each side.
$\sqrt{x+2}=4$	Divide each side by 4.
$\left(\sqrt{x+2}\right)^2=4^2$	Square each side of the equation.
$x+2=16$	Simplify.
$x=14$	Subtract 2 from each side.

▶ The solution is $x = 14$.

EXAMPLE 3 Solving an Equation with Radicals on Both Sides

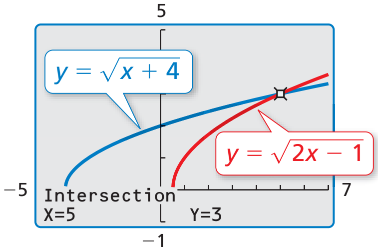
Solve $\sqrt{2x-1}=\sqrt{x+4}$.

SOLUTION

Method 1

$\sqrt{2x-1}=\sqrt{x+4}$	Write the equation.
$\left(\sqrt{2x-1}\right)^2=\left(\sqrt{x+4}\right)^2$	Square each side of the equation.
$2x-1=x+4$	Simplify.
$x=5$	Solve for x.

▶ The solution is $x = 5$.



Method 2 Graph each side of the equation, as shown. Use the *intersect* feature to find the coordinates of the point of intersection. The x-value of the point of intersection is 5.

▶ So, the solution is $x = 5$.

EXAMPLE 4 Solving a Radical Equation Involving a Cube Root

Solve $\sqrt[3]{5x-2}=12$.

SOLUTION

$\sqrt[3]{5x-2}=12$	Write the equation.
$\left(\sqrt[3]{5x-2}\right)^3=12^3$	Cube each side of the equation.
$5x-2=1728$	Simplify.
$x=346$	Solve for x.

▶ The solution is $x = 346$.

LOOKING FOR STRUCTURE

You can extend the concept taught in Examples 1–3 to solve a radical equation involving a cube root. Instead of squaring each side of the equation, you *cube* each side to eliminate the radical.

Monitoring Progress

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Solve the equation. Check your solution.

5. $\sqrt{x+4}+7=11$ **6.** $15=6+\sqrt{3w-9}$ **7.** $\sqrt{3x+1}=\sqrt{4x-7}$

8. $\sqrt{n}=\sqrt{5n-1}$ **9.** $\sqrt[3]{y}-4=1$ **10.** $\sqrt[3]{3c+7}=10$

Identifying Extraneous Solutions

Squaring each side of an equation can sometimes introduce an extraneous solution.

ATTEND TO PRECISION

To understand how extraneous solutions can be introduced, consider the equation $\sqrt{x} = -2$. This equation has no real solution, however, you obtain $x = 4$ after squaring each side.

EXAMPLE 5 Identifying an Extraneous Solution

Solve $x = \sqrt{x + 6}$.

SOLUTION

$x = \sqrt{x+6}$ Write the equation.

$x^2 = \left(\sqrt{x+6}\right)^2$ Square each side of the equation.

$x^2 = x + 6$ Simplify.

$x^2 - x - 6 = 0$ Subtract x and 6 from each side.

$(x - 3)(x + 2) = 0$ Factor.

$x - 3 = 0$ or $x + 2 = 0$ Zero-Product Property

$x = 3$ or $x = -2$ Solve for x.

Check Check each solution in the original equation.

$3 \stackrel{?}{=} \sqrt{3+6}$	Substitute for x.	$-2 \stackrel{?}{=} \sqrt{-2+6}$
$3 \stackrel{?}{=} \sqrt{9}$	Simplify.	$-2 \stackrel{?}{=} \sqrt{4}$
$3 = 3$ ✓	Simplify.	$-2 \neq 2$ ✗

STUDY TIP

Be sure to always substitute your solutions into the original equation to check for extraneous solutions.

▶ Because $x = -2$ does not satisfy the original equation, it is an extraneous solution. The only solution is $x = 3$.

EXAMPLE 6 Identifying an Extraneous Solution

Solve $13 + \sqrt{5n} = 3$.

SOLUTION

$13 + \sqrt{5n} = 3$ Write the equation.

$\sqrt{5n} = -10$ Subtract 13 from each side.

$\left(\sqrt{5n}\right)^2 = (-10)^2$ Square each side of the equation.

$5n = 100$ Simplify.

$n = 20$ Divide each side by 5.

Check

$13 + \sqrt{5n} = 3$

$13 + \sqrt{5(20)} \stackrel{?}{=} 3$

$13 + \sqrt{100} \stackrel{?}{=} 3$

$23 \neq 3$ ✗

▶ Because $n = 20$ does not satisfy the original equation, it is an extraneous solution. So, the equation has no solution.

Monitoring Progress

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Solve the equation. Check your solution(s).

11. $\sqrt{4 - 3x} = x$ **12.** $\sqrt{3m} + 10 = 1$ **13.** $p + 1 = \sqrt{7p + 15}$

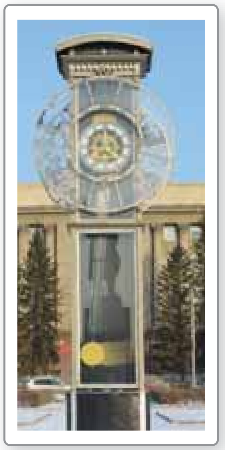
Solving Real-Life Problems

EXAMPLE 7 Modeling with Mathematics

STUDY TIP

The period of a pendulum is the amount of time it takes for the pendulum to swing back and forth.

The period P (in seconds) of a pendulum is given by the function $P = 2\pi\sqrt{\frac{L}{32}}$, where L is the pendulum length (in feet). A pendulum has a period of 4 seconds. Is this pendulum twice as long as a pendulum with a period of 2 seconds? Explain your reasoning.



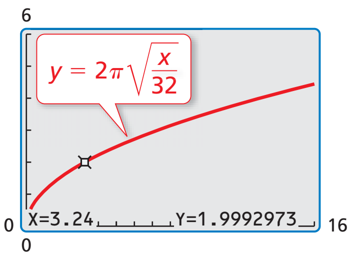
SOLUTION

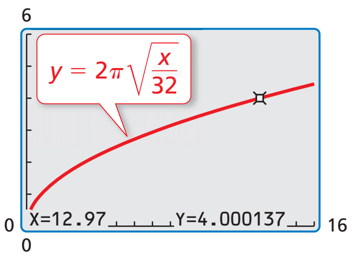
1. **Understand the Problem** You are given a function that represents the period P of a pendulum based on its length L. You need to find and compare the values of L for two values of P.
2. **Make a Plan** Substitute $P = 2$ and $P = 4$ into the function and solve for L. Then compare the values.
3. **Solve the Problem**

$P = 2\pi\sqrt{\frac{L}{32}}$	Write the function.	$P = 2\pi\sqrt{\frac{L}{32}}$
$2 = 2\pi\sqrt{\frac{L}{32}}$	Substitute for P.	$4 = 2\pi\sqrt{\frac{L}{32}}$
$\frac{2}{2\pi} = \sqrt{\frac{L}{32}}$	Divide each side by 2π.	$\frac{4}{2\pi} = \sqrt{\frac{L}{32}}$
$\frac{1}{\pi} = \sqrt{\frac{L}{32}}$	Simplify.	$\frac{2}{\pi} = \sqrt{\frac{L}{32}}$
$\frac{1}{\pi^2} = \frac{L}{32}$	Square each side and simplify.	$\frac{4}{\pi^2} = \frac{L}{32}$
$\frac{32}{\pi^2} = L$	Multiply each side by 32.	$\frac{128}{\pi^2} = L$
$3.24 \approx L$	Use a calculator.	$12.97 \approx L$

▶ No, the length of the pendulum with a period of 4 seconds is $\frac{128}{\pi^2} \div \frac{32}{\pi^2} = 4$ times longer than the length of a pendulum with a period of 2 seconds.

4. **Look Back** Use the *trace* feature of a graphing calculator to check your solutions.





Monitoring Progress

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14. What is the length of a pendulum that has a period of 2.5 seconds?

10.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Why should you check every solution of a radical equation?

2. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

$\sqrt{x} + 6 = 10$	$2\sqrt{x + 3} = 32$	$x\sqrt{3} - 5 = 4$	$\sqrt{x - 1} = 16$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–12, solve the equation. Check your solution. *(See Example 1.)*

3. $\sqrt{x} = 9$

4. $\sqrt{y} = 4$

5. $7 = \sqrt{m} - 5$

6. $\sqrt{p} - 7 = -1$

7. $\sqrt{c} + 12 = 23$

8. $\sqrt{x} + 6 = 8$

9. $4 - \sqrt{a} = 2$

10. $-8 = 7 - \sqrt{r}$

11. $3\sqrt{y} - 18 = -3$

12. $2\sqrt{q} + 5 = 11$

In Exercises 13–20, solve the equation. Check your solution. *(See Example 2.)*

13. $\sqrt{a - 3} + 5 = 9$

14. $\sqrt{b + 7} - 5 = -2$

15. $2\sqrt{x + 4} = 16$

16. $5\sqrt{y - 2} = 10$

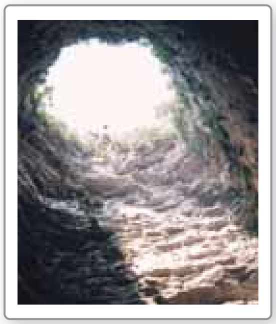
17. $-1 = \sqrt{5r + 1} - 7$

18. $2 = \sqrt{4s - 4} - 4$

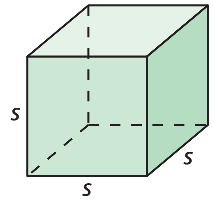
19. $7 + 3\sqrt{3p - 9} = 25$

20. $19 - 4\sqrt{3c - 11} = 11$

21. **MODELING WITH MATHEMATICS** The Cave of Swallows is a natural open-air pit cave in the state of San Luis Potosí, Mexico. The 1220-foot-deep cave was a popular destination for BASE jumpers. The function $t = \frac{1}{4}\sqrt{d}$ represents the time t (in seconds) that it takes a BASE jumper to fall d feet. How far does a BASE jumper fall in 3 seconds?

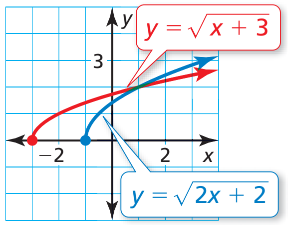


22. **MODELING WITH MATHEMATICS** The edge length s of a cube with a surface area of A is given by $s = \sqrt{\frac{A}{6}}$. What is the surface area of a cube with an edge length of 4 inches?

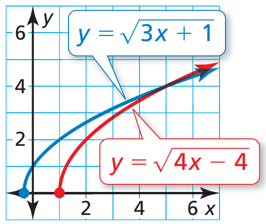


In Exercises 23–26, use the graph to solve the equation.

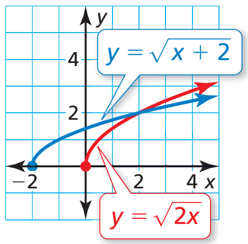
23. $\sqrt{2x + 2} = \sqrt{x + 3}$



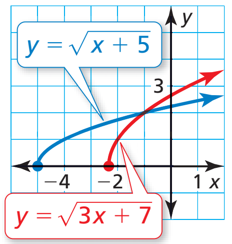
24. $\sqrt{3x + 1} = \sqrt{4x - 4}$



25. $\sqrt{x + 2} - \sqrt{2x} = 0$



26. $\sqrt{x + 5} - \sqrt{3x + 7} = 0$



In Exercises 27–34, solve the equation. Check your solution. *(See Example 3.)*

27. $\sqrt{2x - 9} = \sqrt{x}$

28. $\sqrt{y + 1} = \sqrt{4y - 8}$

29. $\sqrt{3g + 1} = \sqrt{7g - 19}$

30. $\sqrt{8h - 7} = \sqrt{6h + 7}$

31. $\sqrt{\frac{p}{2} - 2} = \sqrt{p - 8}$

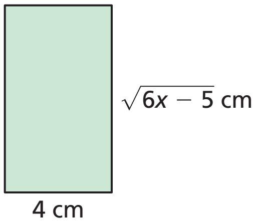
32. $\sqrt{2v - 5} = \sqrt{\frac{v}{3} + 5}$

33. $\sqrt{2c + 1} - \sqrt{4c} = 0$

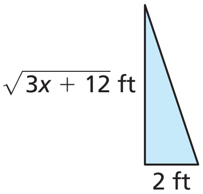
34. $\sqrt{5r} - \sqrt{8r - 2} = 0$

MATHEMATICAL CONNECTIONS **In Exercises 35 and 36, find the value of x.**

35. Perimeter = 22 cm



36. Area = $\sqrt{5x-4}$ ft^2



In Exercises 37–44, solve the equation. Check your solution. *(See Example 4.)*

37. $\sqrt[3]{x} = 4$

38. $\sqrt[3]{y} = 2$

39. $6 = \sqrt[3]{8g}$

40. $\sqrt[3]{r+19} = 3$

41. $\sqrt[3]{2s+9} = -3$

42. $-5 = \sqrt[3]{10x+15}$

43. $\sqrt[3]{y+6} = \sqrt[3]{5y-2}$

44. $\sqrt[3]{7j-2} = \sqrt[3]{j+4}$

In Exercises 45–48, determine which solution, if any, is an extraneous solution.

45. $\sqrt{6x-5} = x$; $x = 5$, $x = 1$

46. $\sqrt{2y+3} = y$; $y = -1$, $y = 3$

47. $\sqrt{12p+16} = -2p$; $p = -1$, $p = 4$

48. $-3g = \sqrt{-18-27g}$; $g = -2$, $g = -1$

In Exercises 49–58, solve the equation. Check your solution(s). *(See Examples 5 and 6.)*

49. $y = \sqrt{5y-4}$

50. $\sqrt{-14-9x} = x$

51. $\sqrt{1-3a} = 2a$

52. $2q = \sqrt{10q-6}$

53. $9 + \sqrt{5p} = 4$

54. $\sqrt{3n} - 11 = -5$

55. $\sqrt{2m+2} - 3 = 1$

56. $15 + \sqrt{4b-8} = 13$

57. $r + 4 = \sqrt{-4r-19}$

58. $\sqrt{3-s} = s - 1$

ERROR ANALYSIS **In Exercises 59 and 60, describe and correct the error in solving the equation.**

59. ✗

$$2 + 5\sqrt{x} = 12$$
$$5\sqrt{x} = 10$$
$$5x = 100$$
$$x = 20$$

60. ✗

$$x = \sqrt{12-4x}$$
$$x^2 = 12 - 4x$$
$$x^2 + 4x - 12 = 0$$
$$(x-2)(x+6) = 0$$
$$x = 2 \quad \text{or} \quad x = -6$$

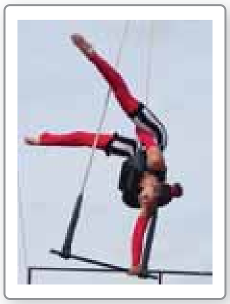
The solutions are $x = 2$ and $x = -6$.

61. **REASONING** Explain how to use mental math to solve $\sqrt{2x} + 5 = 1$.

62. **WRITING** Explain how you would solve $\sqrt[4]{m+4} - \sqrt[4]{3m} = 0$.

63. **MODELING WITH MATHEMATICS** The formula $V = \sqrt{PR}$ relates the voltage V (in volts), power P (in watts), and resistance R (in ohms) of an electrical circuit. The hair dryer shown is on a 120-volt circuit. Is the resistance of the hair dryer half as much as the resistance of the same hair dryer on a 240-volt circuit? Explain your reasoning. *(See Example 7.)*

64. **MODELING WITH MATHEMATICS** The time t (in seconds) it takes a trapeze artist to swing back and forth is represented by the function $t = 2\pi\sqrt{\frac{r}{32}}$, where r is the rope length (in feet). It takes the trapeze artist 6 seconds to swing back and forth. Is this rope $\frac{3}{2}$ as long as the rope used when it takes the trapeze artist 4 seconds to swing back and forth? Explain your reasoning.



REASONING **In Exercises 65–68, determine whether the statement is *true* or *false*. If it is false, explain why.**

65. If $\sqrt{a} = b$, then $(\sqrt{a})^2 = b^2$.

66. If $\sqrt{a} = \sqrt{b}$, then $a = b$.

67. If $a^2 = b^2$, then $a = b$.

68. If $a^2 = \sqrt{b}$, then $a^4 = (\sqrt{b})^2$.

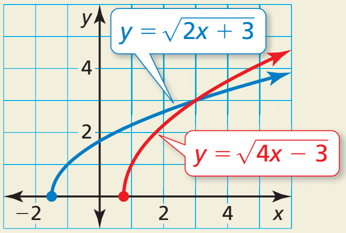
69. **COMPARING METHODS** Consider the equation $x + 2 = \sqrt{2x - 3}$.

a. Solve the equation by graphing. Describe the process.

b. Solve the equation algebraically. Describe the process.

c. Which method do you prefer? Explain your reasoning.

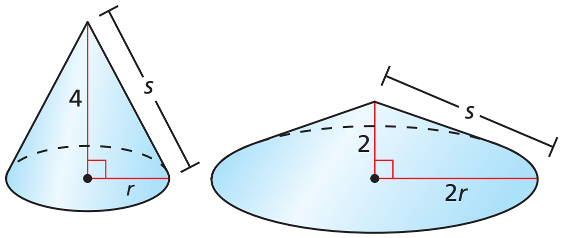
70. **HOW DO YOU SEE IT?** The graph shows two radical functions.



a. Write an equation whose solution is the x-coordinate of the point of intersection of the graphs.

b. Use the graph to solve the equation.

71. **MATHEMATICAL CONNECTIONS** The slant height s of a cone with a radius of r and a height of h is given by $s = \sqrt{r^2 + h^2}$. The slant heights of the two cones are equal. Find the radius of each cone.



72. **CRITICAL THINKING** How is squaring $\sqrt{x + 2}$ different from squaring $\sqrt{x} + 2$?

USING STRUCTURE **In Exercises 73–78, solve the equation. Check your solution.**

73. $\sqrt{m + 15} = \sqrt{m} + \sqrt{5}$

74. $2 - \sqrt{x + 1} = \sqrt{x + 2}$

75. $\sqrt{5y + 9} + \sqrt{5y} = 9$

76. $\sqrt{2c - 8} - \sqrt{2c} - 4 = 0$

77. $2\sqrt{1 + 4h} - 4\sqrt{h} - 2 = 0$

78. $\sqrt{20 - 4z} + 2\sqrt{-z} = 10$

79. **OPEN-ENDED** Write a radical equation that has a solution of $x = 5$.

80. **OPEN-ENDED** Write a radical equation that has $x = 3$ and $x = 4$ as solutions.

81. **MAKING AN ARGUMENT** Your friend says the equation $\sqrt{(2x + 5)^2} = 2x + 5$ is always true, because after simplifying the left side of the equation, the result is an equation with infinitely many solutions. Is your friend correct? Explain.

82. **THOUGHT PROVOKING** Solve the equation $\sqrt[3]{x + 1} = \sqrt{x - 3}$. Show your work and explain your steps.

83. **MODELING WITH MATHEMATICS** The frequency f (in cycles per second) of a string of an electric guitar is given by the equation $f = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$, where ℓ is the length of the string (in meters), T is the string's tension (in newtons), and m is the string's mass per unit length (in kilograms per meter). The high E string of an electric guitar is 0.64 meter long with a mass per unit length of 0.000401 kilogram per meter.

a. How much tension is required to produce a frequency of about 330 cycles per second?

b. Would you need more or less tension to create the same frequency on a string with greater mass per unit length? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the product. *(Section 7.2)*

84. $(x + 8)(x - 2)$

85. $(3p - 1)(4p + 5)$

86. $(s + 2)(s^2 + 3s - 4)$

Graph the function. Compare the graph to the graph of $f(x) = x^2$. *(Section 8.1)*

87. $r(x) = 3x^2$

88. $g(x) = \frac{3}{4}x^2$

89. $h(x) = -5x^2$